than 2 or are a product of prime numbers. Then $n\dfrac{(n+1)}{2}$ cannot be two times one prime number.

The triangular numbers that are twice a prime must come from positive integers n which are not greater than 4. We see that the triangular numbers 6 when $n = 3$ and 10 when $n = 4$ are the only triangular numbers which are twice a prime number.

Also solved by Dionne Bailey, Elsie Campbell and Charles Diminnie, Angelo State University, San Angelo, TX; Brian D. Beasely, Presbyterian College, Clinton, SC; Jahangeer Kholdi and Farideh Firoozbakht, University of Isfahan, Khansar, Iran; Kenneth Korbin, New York, NY; Kee-Wai Lau, Hong Kong, China; David E. Manes, SUNY College at Oneonta, Oneonta, NY; Neculai Stanciu and Titu Zvonaru, Romania; David Stone and John Hawkins, Georgia Southern University, Statesboro, GA, and the proposer.

- **5279:** *Proposed by D.M. Bătinetu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" General School, Buzău, Romania*

Let $f : \Re_+ \longrightarrow \Re_+$ be a convex function on $\Re_+$, where $\Re_+$ stands for the positive real numbers. Prove that

$$3\left(f^2(x)+f^2(y)+f^2(z)\right)-9f^2\left(\frac{x+y+z}{3}\right) \geq (f(x)-f(y))^2+(f(y)-f(z))^2+(f(z)-f(x))^2 .$$

Solution 1 by Arkady Alt, San Jose, CA

Since

$$\begin{aligned} & 3\left(f^2(x)+f^2(y)+f^2(z)\right)-(f(x)-f(y))^2+(f(y)-f(z))^2+(f(z)-f(x))^2 \\ = \; & (f(x)+f(y)+f(z))^2 , \end{aligned}$$

the original inequality is equivalent to

$$(f(x)+f(y)+f(z))^2 \geq 9f^2\left(\frac{x+y+z}{3}\right) \iff \frac{f(x)+f(y)+f(z)}{3} \geq f\left(\frac{x+y+z}{3}\right),$$

where the latter inequality is Jensen's Inequality for the convex function $f : \Re_+ \longrightarrow \Re_+$.

Solution 2 by Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain

Since f is convex, then $f\left(\dfrac{x+y+z}{3}\right) \leq \dfrac{f(x)+f(y)+f(z)}{3}$ and the left-hand side of the given inequality is

$$\begin{aligned} LHS &\geq 3\left(f^2(x)+f^2(y)+f^2(z)\right)-(f(x)+f(y)+f(z))^2 \\ &= 2\left(f^2(x)+f^2(y)+f^2(z)\right)-(2f(x)f(y)+2f(y)f(z)+2f(z)f(x)) \\ &= (f(x)-f(y))^2+(f(y)-f(z))^2+(f(z)-f(x))^2 . \end{aligned}$$

Solution 3 by Michael Brozinsky, Central Islip, NY